Topics for Qualifying Exam in Complex Analysis

I Complex Plane and Elementary Function.
- a) Complex Numbers
- b) Polar Representation
- c) Stereographic Projection
- d) The Square and Square Root Functions
- e) The Exponential Function
- f) The Logarithm Function
- g) Power Functions and Phase Factors
- h) Trigonometric and Hyperbolic Functions

II Analytic Functions
- a) Review of Basic Analysis
- b) Analytic Functions
- c) The Cauchy-Riemann Equations
- d) Inverse Mappings and the Jacobian
- e) Harmonic Functions
- f) Conformal Mappings
- g) Fractional Linear Transformations

III Line Integrals and Harmonic Functions
- a) Line Integrals and Green's Theorem
- b) Independence of Path
- c) Harmonic Conjugates
- d) The Mean Value Property
- e) The Maximum Principle

IV Complex Integration and Analyticity
- a) Complex Line Integrals
- b) Fundamental Theorem of Calculus for Analytic Functions
- c) Cauchy's Theorem
- d) The Cauchy Integral Formula
- e) Liouville's Theorem
- f) Morera's Theorem
- g) Goursat's Theorem
- h) Complex Notation and Pompeiu's Formula

V Power Series
- a) Infinite Series
- b) Sequences and Series of Functions
- c) Power Series
- d) Power Series Expansion of an Analytic Function
- e) Power Series Expansion at Infinity
- f) Manipulation of Power Series
- g) The Zeros of an Analytic Function

h) Analytic Continuation

VI Laurent Series and Isolated Singularities
- a) The Laurent Decomposition
- b) Isolated Singularities of an Analytic Function
- c) Isolated Singularity at Infinity
- d) Partial Fractions Decomposition

VII The Residue Calculus
- a) The Residue Theorem
- b) Integrals Featuring Rational Functions
- c) Integrals of Trigonometric Functions
- d) Integrands with Branch Points
- e) Fractional Residues
- f) Principal Values
- g) Jordan's Lemma
- h) Exterior Domains

VIII The Logarithmic Integral
- a) The Argument Principle
- b) Rouche's Theorem
- c) Hurwitz's Theorem
- d) Open Mapping and Inverse Function Theorems

IX The Schwarz Lemma and Hyperbolic Geometry
- a) The Schwarz Lemma
- b) Conformal Self-Maps of the Unit Disk

X Harmonic Functions and the Reflection Principle
- a) The Poisson Integral Formula
- b) Characterization of Harmonic Functions
- c) The Schwarz Reflection Principle

XI Conformal Mapping
- a) Mappings to the Unit Disk and Upper Half-Plane
- b) The Riemann Mapping Theorem
- c) Compactness of Families of Functions
- d) Proof of the Riemann Mapping Theorem

<u>References: Complex Analysis by T.W. Gamelin</u>