Topics for Qualifying Exam in Combinatorics

I. Enumeration
- A. Selections with and without repetitions (combinations & permutations)
- B. Partitions
  1. Stirling numbers of the first and second kind
- C. Principle of Inclusion-Exclusion
  1. Surjections
  2. Derangements
  3. Euler ϕ function
- D. Generating functions (ordinary and exponential)
  1. For combinations and permutations
  2. Solutions of recurrence relations
  3. Catalan numbers
- E. Pólya theory

II. Combinatorial designs
- A. Design parameters
- B. Balanced incomplete block designs (BIBDs)
- C. Partially-balanced incomplete balanced designs (PBIBDs)
- D. Triple systems
- E. Resolvable designs
- F. Finite geometries
  1. Affine geometries
  2. Projective geometries
  3. Partial geometries
- G. Latin squares
  1. Orthogonality

III. Matroids
- A. Independence, basis, spanning, and cycle systems; rank function and closure operator
  1. The definition in terms of each of these
  2. Proofs of equivalences of these definitions
- B. Duality
- C. Representability
- D. Greedy algorithm

IV. Pigeonhole principle and Ramsey's theorem

V. Minimax theory
   A. Max-flow-min-cut theorem
   B. Optimization algorithms
   C. Menger's theorem (see VI.B.2)
   D. P. Hall Theorem (marriage-, matching-problems, etc.)
   E. {0,1}-Matrices

VI. Graph theory
   A. Basic notions
      1. Euler paths/circuits
      2. Hamilton paths/circuits
      3. Adjacency and incidence matrices
         a. Characteristic polynomials and graph spectra
      4. Forests and trees
      5. Graph products (cartesian, categorical, weak, lexicogrphaphic)
      6. Cycle and cocycle spaces
   B. Connectivity
      1. Biconnectivity
      2. Menger's and Whitney's characterization of *m*-connectivity
      3. Atoms and fragments
   C. Planarity
      1. Euler's formula
      2. Kuratowski's theorem
      3. Tilings
   D. Topological graph theory
      1. Edmond's embedding theorem
      2. Heawood's theorem
   E. Chromatic graph theory
      1. Color-critical graphs
      2. Four-color theorem
      3. Edge-colorings
   F. Algebraic graph theory
      1. Vertex- and edge-automorphism groups
      2. Arc-transitivity
      3. Distance-regular and distance-transitive graphs
      4. Cayley graphs
      5. Properties of special graphs
         a. *n*-Cages (Heawood graph)
         b. Johnson graphs
         c. Coxeter graph
      6. Automorphism groups of graph products

You should be able to state all definition and basic results for the topics in the above list. You should be able to construct examples that illustrate these definitions and results. You should be able to apply these results to solve specific problems. Finally, you should be prepared to prove any of basic results that have reasonably short proofs. Specifically you will not be expected to prove:

- Vizing's theorem
- Max-flow-min-cut theorem
- Menger's theorem
- Heawood's theorem
- Four-color theorem
- Hamilton path theorems
- Kuratowski's theorem
- Ramsey's theorem

All of the material in this out line can be found in one or more of the following:

References

E.F. Beckenbach (editor), *Applied Combinatorial Mathematics,* (Chapters 3, 5, 12, 13)
J.A. Bondy & U.S.R. Murty, *Graph Theory with Applications*
R. A. Brualdi, *Introductory Combinatorics*
R. Diestel, *Graph Theory,* (Chapters 1 - 6, 10)
C.D. Godsil & G. Royle, *Algebraic Graph Theory,* (Chapters 1, 2, 3, 4, 6)
J.E. Graver & M.E. Watkins, *Combinatorics with Emphasis on the Theory of Graphs*
W. Imrich & S. Klavzar, *Product Graphs,* (Chapters 1, 4, 5, 6)
C.L. Liu, *Introduction to Combinatorial Mathematics,* (all but Chapters 12 & 13)
R.P. Stanley, *Enumerative Combinatorics,* (Volume I)
H.J. Ryser, *Combinatorial Mathematics,* (all but Chapter 9)