Syllabus for MAT 526 – Summer Online Version

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Class: This class is taken entirely online via the class *Blackboard* site. Access to some other equipment and sofware is required (see below). No visits to the SU campus are required.

Text: Introduction to Probability Models, 11th Edition, by Sheldon M. Ross, Elsevier, Amsterdam, 2014. (Print or Electronic version)

Other required: Access to *Skype*, capability to produce jpg images and short mp4 video clips. Access to the internet, including email.

Catalogue Description:

> MAT 526 **Introduction to Stochastic Processes** 3 S Discrete time Markov chains, Poisson process, continuous time Markov chains and other selected stochastic processes. PREREQ: MAT 521 or graduate standing in mathematical sciences

Course Description:

Make sure you read this section thoroughly, and understand it completely.

Please note that the catalogue description allows some flexibility in the choice of subject matter. This course will concentrate on 4 specific probability models: (i) Discrete parameter Markov chains with finite or countable state space, (ii) Poisson processes, (iii) Continuous time Markov processes, (iv) Brownian motion. Each will be illustrated with one or more examples drawn from or motivated by applications, although the emphasis will be on mathematical issues and on obtaining a broad introduction to the main ideas and results in each area.

The course is divided into 9 weeks. Generally, each week has two modules, but these only serve to group the written and video assignments. Each module comprises:

- A set of recorded video lectures
- Quizzes accompany many lectures to help you check your understanding.
- A set of “routine” exercises with available solutions, for study

- A set of assigned problems, to be submitted electronically for grading, generally each Tuesday and Friday of each week. Often there will be one problem solution to be submitted as a video on Friday.
- Assigned reading
- Participation in online QandA sessions, to be held each Monday and Thursday.

In a typical week of work you should pace your work in order to keep up without getting swamped. You should always view the lectures and keep a notebook of questions you may have about them. There are two modules per week and 4 lectures per module (one or two will often be marked optional.) So about one lecture per day (after the first week) should be enough, but you need to be disciplined about watching them. Don't worry too much about understanding absolutely everything the first time through. You can always rewatch as many times as you need to. When you are satisfied that you fully understand a lecture, try the quiz that goes along with it, if there is one. (The quizzes are not counted towards the grade – they are for your information.)

As you work on viewing the videos also do the assigned reading. Again, it is not necessary to understand absolutely everything the first time through. I almost always read mathematics at least twice. The last time through, my goal is to understand absolutely everything. Make margin notes or underline things you have trouble with to ask about later. The margin is one of the most useful parts of a print book. (Assuming you own it.)

Next, try to work the example exercises yourself. The textbook lacks routine exercises. The intent of the example worked exercises is to make up for this lack. Treat these as exercises: look at the list of problems and try to work them yourself before you look at the answers. Note that the starred problems in the text have answers in the back of the book, so trying to work some of these is another way to test your understanding. Word to the wise: you may well be asked to work some of the example exercises on oral exams.

Only begin the assigned problems (from the text) when you feel you are ready for a slightly more challenging question that will usually require some thought about "set up". Review the lectures and the book as you find necessary. Participate in the twice weekly question sessions.

Your homework should be submitted electronically via the *Blackboard* site before 4:30 pm EST on the day it is due (refer to schedule below.) Solutions should be clear and polished, and will be judged, in part, on that basis. They must be in a form that is clearly legible. For example, you could send snapshot photos of legible hand-written pages. Files in pdf format are always usable. You may use whatever computational tools are appropriate and within your skill set. Hand calculation and use of a symbolic and graphic calculator is fine, as long as your writeup allows me to follow what you did and why. You can even use computer software, but you'd better make it produce results in a form I can interpret. I can handle office documents, plain text, maple work sheets, postscript files, pdfs, and gif or jpg image files. That's about it. Check with me if you feel you absolutely must use something that is not in one of these forms. Quite frankly, everything should be quite doable with just the help of a scientific calculator, writing the solution on a piece of paper, and submitting an electronic photo.

Most weeks I will ask you to submit the solution of one problem as an mp4 video clip. You will need to find a whiteboard or blackboard in a room where you can talk out load and have somebody record a video of you (say, on a smartphone). You could even do a “selfie.” Submit these just like you do written homework.

DO NOT mine the internet for solutions to problems. You don’t learn anything by doing this, and, what’s more, it’s a violation of academic integrity unless you clearly credit any outside sources. Indeed, that goes for all work submitted as your own: if you must use someone else’s ideas, you MUST give that person or organization due credit.

Grades on assignments will be posted on *Blackboard*. You will not necessarily receive other feedback on assignments (unless you request it), and if you do, it will be normally be by email.

You will also be required to participate in 3 one-on-one *Skype* sessions with the instructor. Sign up for these on the blackboard site. They will last 15 minutes each, and will be held the first week, the fifth week, and the last week. The first one is just an informal interview for me to meet you and answer any questions you may have about the course, and to work out any “bugs” in the connection process. The one in week 5 will function as a mid-term oral exam, and the one in the last week as an oral final exam.

A detailed outline of the various units is given at the end of this syllabus. All course materials (except for the textbook) are available from the course *Blackboard* site.

Learning goals for this course: The successful student will be able to

- Choose the most appropriate mathematical model for a given real world application from among those studied
- Critically evaluate applicability of the chosen model
- Reduce most questions of interest about a selected model to probability theory calculations
- Accurately calculate relevant probabilities using an appropriate tool, ranging from hand calculation to symbolic software packages
- Locate information about more sophisticated models when needed

Math Department Learning Outcomes Mapped to this Course:

- Demonstrate facility with the techniques of single and multivariable calculus and linear algebra
- Effectively communicate mathematical ideas
- Make accurate calculations by hand and with technological assistance
- Reproduce essential assumptions, definitions, examples, and statements of important theorems
- Describe the logical structure of the standard proof formats, reproduce the underlying ideas of the proofs of basic theorems, and create simple original proofs.
- Solve standard science and engineering problems by selecting and applying an appropriate mathematical model

- Demonstrate depth of understanding in one of the following areas: differential and linear equations, analysis, probability and statistics

Prerequisite: MAT 521, or equivalent calculus-based semester-long introduction to probability.

Grading: A course grade number (percent) will be obtained by weighting the written homework 30%, the video homework 30%, the midterm oral 20%, and the final oral 20%. The resulting percentage will then be rounded up to the next whole number and the letter grade will be determined as in the following table:

Grades	Grade Number (percent)
A	93-100
A-	90-92
B+	87-89
B	83-86
B-	80-82
C+	77-79
C	73-76
C-	70-72
D [1]	65-69
D- [1,2]	60-64
F	Below 60

[1] Grades of D and D- may not be assigned to graduate students.
[2] Available only for Law students in LAW courses.

Academic Integrity Policy

Syracuse University's academic integrity policy reflects the high value that we, as a university community, place on honesty in academic work. The policy defines our expectations for academic honesty and holds students accountable for the integrity of all work they submit. Students should understand that it is their responsibility to learn about course-specific expectations, as well as about university-wide academic integrity expectations. The university policy governs appropriate citation and use of sources, the integrity of work submitted in exams and assignments, and the veracity of signatures on attendance sheets and other verification of participation in class activities. The policy also prohibits students from submitting the same written work in more than one class without receiving written authorization in advance from both instructors. The presumptive penalty for a first instance of academic dishonesty by an undergraduate student is course failure, accompanied by a transcript notation indicating that the failure resulted from a violation of academic integrity policy. The presumptive penalty for a first instance of academic dishonesty by a graduate student is suspension or expulsion. SU students are required to read an online summary of the university's academic integrity expectations and provide an electronic signature agreeing to abide by them twice a year during pre-term check-in on MySlice. For more information and the complete policy, see http://academicintegrity.syr.edu/

Disability-Related Accommodations

If you believe that you need accommodations for a disability, please contact the Office of Disability Services (ODS), http://disabilityservices.syr.edu, located in Room 309 of 804 University Avenue, or call (315) 443-4498, TDD: (315) 443-1371 for an appointment to discuss your needs and the process for requesting accommodations. ODS is responsible for coordinating disability-related accommodations and will issue students with documented Disabilities Accommodation Authorization Letters, as appropriate. Since accommodations may require early planning and generally are not provided retroactively, please contact ODS as soon as possible.

Religious Observances Notification and Policy

SU religious observances notification and policy, found at http://hendricks.syr.edu/spiritual-life/index.html, recognizes the diversity of faiths represented among the campus community and protects the rights of students, faculty, and staff to observe religious holidays according to their tradition. Under the policy, students are provided an opportunity to make up any examination, study, or work requirements that may be missed due to a religious observance provided they notify their instructors before the end of the second week of classes for regular session classes and by the submission deadline for flexibly formatted classes.

For fall and spring semesters, an online notification process is available for students in **My Slice / StudentServices / Enrollment / MyReligiousObservances / Add a Notification.** Instructors may access a list of their students who have submitted a notification in My Slice Faculty Center.

Course Outline and Schedule

- **Week 1**

Module 1: What is a Stochastic Process?

Overview: In this module we will review some important topics from probability theory and introduce the concept of stochastic process

Learning Objectives: By the end of this module you will have completed a review of the following topics:

Probability models
Calculation of random variable (and random vector) probabilities of interest from the (joint) pdf or pf
Calculation of and meaning of moments
Conditional probabilities, partitioning formula, and Bayes’ formula

Activities:

Watch introductory and review video lectures
Complete Module Quizzes
Readings: Familiarize yourself with the first 3 chapters and review in greater detail as needed in order to do homework and follow the lectures.
You can return and read parts of these chapters as needed throughout the course
Homework Due First Friday: p.83 #34; p. 90 #86; Video: p. 176 #74

Module 2: Exponential and Poisson Distributions

Overview:
In this module we will discuss the Poisson and Exponential Distributions.

Learning Objectives:
By the end of this module you will be able to …

Determine appropriate parameter values from givens
Calculate exponential and Poisson probabilities and moments
Gauge whether these distributions are appropriate models in a given situation
Use the special properties of these distributions in problem solving

Activities:

Watch Video Lectures on Exponential and Poisson Distributions
Complete Module Quizzes
Readings: 2.2.1-2, 2.2.4; examples 2.19, 2.41, 2.45,2.47; pp74-5; 5.2.1-4
Homework Assignment Due 2nd Tuesday: p342 #26, p343 #30
Worked Exercises (optional, but strongly encouraged)

- **Week 2**

Module 3: Introduction to the Poisson Process

Overview:

In this module we will introduce the Poisson Process and its basic properties

Learning Objectives:
By the end of this module you will be able to...

Determine whether a Poisson Process (PP) model is appropriate in a given problem
Indentify arrival rate(s) from givens
Apply properties of the process in probability calculations
Use distribution laws of increments and inter-arrival times in calculations

Translate between descriptions involving counts and ones involving arrival times

Activities:

Watch Video Lectures on the Poisson Process
Complete Module Quizzes
Readings: 5.3.1 - 5.3.3
Homework Assignment Due 2nd Friday: p. 344 #39, p345 #44 Video: #45
Worked Exercises (optional, but strongly encouraged)

Module 4: Poisson Process (continued)

Overview:

In this module we study further properties of the Poisson Process: scaling, pruning, and the joint conditional distribution of arrival times

Learning Objectives:
By the end of this module you will be able to...

Describe mathematically several operations that give new PPs from old
Identify these operations when they occur in problems/applications
Describe properties of sample paths and explain why they occur
Work with the joint distribution of arrival times conditional on their lying in a given time interval

Activities:

Watch video lectures on Poisson Processes
Complete Module Quizzes
Readings: 5.3.4, 5.3.5
Homework Due 3rd Tuesday: p. 348 #62; p. 350 #72.
Worked Exercises (optional, but strongly encouraged)

- **Week 3**

Module 5: Generalizations of the Poisson Process

Overview:

In this module we will study some generalizations of the Poisson process: non-stationary processes, compound Poisson processes, mixtures, and queueing models

Learning Objectives:
By the end of this module you will be able to...

Solve problems involving non-homogeneous Poisson processes

Solve problems involving compound Poisson processes
Solve problems involving mixtures of Poisson processes
Identify queueing models, and distinguish among the various model types

Activities:

Watch video lectures on Poisson Processes
Complete Module Quizzes
Readings: 5.4.1, 5.4.2 (skip Example 5.27), Examples 5.18, 5.25, and 5.27.
Homework: due 3rd Friday p. 352 #78; p. 353 #86 Video: #85
Worked Exercises (optional, but strongly encouraged)

Module 6: Introduction to Markov Chains

Overview:

In this module we will introduce Markov chain models: state spaces, transition matrices, classification.

Learning Objectives:
By the end of this module you will be able to...

Construct Markov chain models
Calculate chain probabilities in terms of an initial probability vector and transition matrix
Distinguish among various state properties: transience, recurrence, and periodicity
Decompose a chain into communicating classes

Activities:

Watch video lectures on Markov chains
Complete Module Quizzes
Readings: 4.1, 4.2, 4.3 (through example 4.15)
Homework Due 4th Tuesday: p. 261 #5; p. 262 #14;
Worked Exercises (optional, but strongly encouraged)

- **Week 4**

Module 7: Markov Chains - Positive Recurrence

Overview:

In this module we will explore ergodic behavior in Markov chains.

Learning Objectives:

By the end of this module you will be able to...

Classify states of a Markov chain
Relate transience/recurrence to sums of powers of the transition matrix
Set up equations for stationary probabilities and solve them in simple examples
Interpret the meaning of stationary probabilities

Activities:

Watch video lectures on positive recurrence
Complete Module Quiz #7
Readings: finish 4.3, 4.4 through example 4.23, 4.4.1
Homework due 4th Friday: p. 264 #24, p. 265 #33 Video: #35
Worked Exercises (optional, but strongly encouraged)

Module 8: Markov Chains - Transience

Overview:

In this module we will explore phenomena specific to transient Markov chains.

Learning Objectives:
By the end of this module you will be able to...

Set up equations for harmonic functions and solve them in simple examples
Determine which (nearest neighbor) random walks are recurrent and which are transient
Calculate hitting probabilities of states and sets of states
Calculate expected number of visits to a state
Solve the Gambler's Ruin problem

Activities:

Watch video lectures on transient Markov chains
Complete Module Quiz #8
Readings: 4.5.1, 4.6
Homework due 5th Tuesday: p. 270 #56; p. 271 #60
Worked Exercises (optional, but strongly encouraged)

- **Week 5**

Oral Midterm: Selected Applications of Markov Chains

Overview:

In this module we apply Markov chain theory to random text, branching processes, and Markov chain Monte-Carlo.

Learning Objectives:

By the end of this module you will be able to...

Determine whether a given chain is time-reversible and determine limiting probabilities in this case.
Find the extinction probability for a branching process
Construct reversible chains by assigning edge weights in an undirected graph
Describe the main ideas that underlie the Markov chain Monte-Carlo method
Exhibit all of the above during an online interview

Activities:
Watch video lectures on Markov chain theory application
Readings: Example 4.26, 4.7, 4.8, 4.9
Be prepared to discuss these problems in the oral exam: p. 272 #66; p. 273 #76, #72 (These are not to be handed in separately)
Worked Exercises: Be prepared to solve these and any from previous modules in the oral exam as well.

- **Week 6**

Module 10: Continuous Time Markov Chains - Generalities

Overview:

In this module we will introduce Markov processes in continuous time with discrete state spaces.

Learning Objectives:
By the end of this module you will be able to...

State the Markov property in continuous time
Determine the parameters of a given Markov process model
Define the transition function, state its basic properties and its relation with the process parameters
Set up the Kolmogorov forward and backward equations for a given model

Activities:

Watch video lectures on continuous time Markov Processes
Complete Module Quiz #10
Readings: 6.1, 6.2, 6.3 (through example 6.4), 6.4 (skip example 6.11 for now)
Homework due 6th Friday: p. 398 #1; p. 399 #8. Video: *For a Poisson Process with rate lambda, write down the transition matrix for*
(a) the skeletal Markov chain which samples every u units of time; and
(b) the embedded Markov chain.
Is it true in general that a Poisson Process mixture is a Markov process? Completely justify your answer.

Worked Exercises

Module 11: Limiting Probabilities and Birth and Death Processes

Overview:

In this module we will consider determination of limiting probabilities, and study the class of models known as Birth and Death Processes in detail.

Learning Objectives:
By the end of this module you will be able to...

Set up the equations needed to determine limiting probabilities
Completely analyze the two-state model
Determine limiting probabilities for birth and death processes
Find the transition function for a pure birth process
Completely analyze the M/M/1 queuing model as a special birth and death process

Activities:

Watch video lectures on Birth and Death Processes
Complete Module Quiz #11
Readings: Example 6.11-end of 6.4, 6.5 and 6.9
Homework due 7th Tuesday: p. 399 #9; p. 400 #13; p. 402 #25
Worked Exercises (Optional, but strongly encouraged)

- **Week 7**

Module 12: Introduction to Queuing Theory

Overview:

In this module we introduce more formally some of the main ideas of Queuing Theory and consider the M/G/1 and G/M/1 models in some detail.

Learning Objectives:
By the end of this module you will be able to...

State the assumptions governing the various types of queuing models
Identify and define the main quantities of interest in queuing models
Calculate explicitly all relevant quantities for the M/M/1 model
Determine the parameters of the embedded Markov chain in the M/G/1 and G/M/1 models
Set up the equation needed to determine whether a given model is recurrent or transient

Activities:

Watch video lectures on Queuing Theory
Complete Module Quiz #12
Readings: 8.1, 8.2.1, 8.2.2, 8.5.1, 8.5.2, 8.7 (except 8.7.1)
Homework Due 7th Friday: p. 555 #38; p. 557 #46 Video: p 547 #1
Worked Exercises (Optional, but strongly encouraged)

Module 13: Brownian motion - Introduction and Basic Properties

Overview:

In this module we will introduce one-dimensional Brownian motion process.

Learning Objectives:
By the end of this module you will be able to...

State the axioms of Brownian motion
Set up iterated integrals needed to calculate with Brownian finite dimensional distributions
Calculate moments of Brownian positions and increments
Find the probability that Brownian motion will leave one end of an interval before the other
Find probabilities involving the maximum of Brownian motion

Activities:

Watch video lectures on Brownian motion
Complete Module Quiz #13
Readings: 10.1, 10.2
Homework due 8th Tuesday: p. 639 #7; p. 643 #30
Worked Exercises

- **Week 8**

Module 14: Brownian motion - Semi-Groups and Generators

Overview:

In this module we will study reflecting BM and BM with drift and concepts related to their transition semi-groups.

Learning Objectives:
By the end of this module you will be able to...

State definitions and basic properties of reflecting BM and BM with drift
Solve the gambler's ruin problem for BM with drift
Identify the infinitesimal generators of BM and its variants
Determine whether a function is harmonic and explain the significance of the harmonic property
Give the formulas for the transition functions and state their basic properties

Activities:

Watch video lectures on Brownian semi-groups
Complete Module Quiz #14
Readings: (None)
Homework Due 8th Friday: p. 639 #9; p. 641 #18; Video: *Let T be the time to reach 1 for the reflected Brownian motion started from* $0 < x < 1$*. Find the expected value of T in terms of x.*
Worked Exercises (optional but highly recommended).

Module 15: Brownian Motion: Variations

Overview:

In this module we will introduce higher dimensional BM, the Ito integral, and geometric BM
Learning Objectives:
By the end of this module you will be able to…

Give the form of the generator for BM with drift
Define neighborhood recurrent, and say in which dimensions BM is recurrent, neighborhood recurrent, and transient
Say what it means for a function of 2, 3, or more variables to be harmonic
Solve the analogue of gamber's ruin in higher dimensions, and find the limiting probability of hitting spheres
Outline the construction of the Ito integral and list its main properties
Define geometric Brownian motion

Activities:

Watch video lectures on Brownian motion variations
Readings: 10.3, 10.5, 10.6
Homework due 9th Tuesday: p. 643 #28
Worked Exercises: (Optional, but highly recommended)

- **Week 9**

Module 16: Financial Applications and Oral Final Exam

In this module we will study applications of Brownian motion to mathematical models of the stock market. You will also need to sign up for an oral final exam slot (15 minutes) on *Blackboard*.

Learning Objectives:
By the end of this module you will be able to …

Define arbitrage and illustrate the possibility of arbitrage with examples
Explain the differences among some of the major kinds of stock options
Describe the market model in which the Black-Scholes option pricing formula is valid
Explain, in outline form, how the Black-Scholes equation is derived from the no-arbitrage assumption

Activities

Watch video lectures on finance applications
Readings: 10.4
Oral Exam: Be ready to discuss p 640 #12 and #14. (These are not to be handed in separately).
Also be ready to present the solution of any of the worked exercises since the midterm exam

End of Course ☺